PERIODIC AND QUASI-PERIODIC SOLUTIONS OF 1-D Q-CURVATURE EQUATION

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We consider the following 4-th order ODE

\[ u^{iv} + 10u'' + 9u = \frac{9}{u^2}, \]

the quantity \( Q_g = \frac{1}{9} u^\frac{2}{5} (u^{iv} + 10u'' + 9u) \) is called Q-curvature of \((S^1, g)\) with \( g = u^{-\frac{4}{5}} g_0 \), \( g_0 \) being the standard metric and \( u \) being a positive function defined on \( S^1 \). It is well known that for \( \lambda \in (0, 1], \theta \in [0, \pi) \),

\[ u_{\lambda, \theta}(x) = (\lambda^{-2} \cos^2(x - \theta) + \lambda^2 \sin^2(x - \theta))^\frac{2}{5} \]

are \( \pi \)-periodic solutions of (1), and these are the all \( \pi \)-periodic and \( 2\pi \)-periodic solutions of (1).

In this talk we will discuss other types of solutions, such as periodic and quasi-periodic solutions of (1) via elementary methods. If time permits, we will also discuss existence of solutions of the prescribed Q-curvature equation

\[ u^{iv} + 10u'' + 9u = \frac{9a(x)}{u^2}, \quad x \in S^1 \]

where \( a \) is a positive function defined on \( S^1 \).

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